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CONTACT DISCONTINUITIES IN MODELS OF CONTACT BINARIES UNDERGOING THERMAL RELAXATION OSCILLATIONS

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ABSTRACT

In this paper we pursue the suggestion by Shu, Lubow & Anderson (1979) and Wang (1995) that contact discontinuity (DSC) may exist in the secondary in the expansion TRO (thermal relaxation oscillation) state. It is demonstrated that there is a mass exchange instability in some range of mass ratio for the two components. We show that the assumption of *constant* volume of the secondary should be relaxed in DSC model. For *all* mass ratio the secondary alway satisfies the condition that no mass flow returns to the primary through the inner Lagrangian point. The secondary will expand in order to equilibrate the interaction between the common convective envelope and the secondary. The contact discontinuity in contact binary undergoing thermal relaxation does not violate the second law of thermodynamics. The maintaining condition of contact discontinuity is derived in the time-dependent model. It is desired to improve the TRO model with the advanced contact discontinuity layer in future detailed calculations.

Subject headings: star: contact binary - general

1. INTRODUCTION

W Ursae Majoris (W UMa) binary stars were first thought to be a particular binary population due to their abnormal mass-radius relationship, namely, the so-called Kuiper's paradox, $R_2/R_1 = (M_2/M_1)^{0.46}$ (Kuiper 1941). These particular binaries appear to consist of two main-sequence stars that possess photospheres exhibiting the almost same effective temperatures for the two components despite the fact that typical mass ratio in a system is 0.5. It was originally proposed that a common convective envelope may be formed due to dynamic equilibrium (Osaki 1965), and mass and energy transfers would take place in CCE in order to interpret the Kuiper's paradox (Lucy 1968) although the specific mechanism of energy transfer for the circulation has not been fully understood (Robertson 1980, Sinjab, Robertson & Smith 1990, Tassoul 1992). It is now firmly believed that W UMa stars are contact binaries in which both components are full of their Roche lobe, showing strong interactions (Mochnacki 1981). Lucy's iso-entropy model (1968) as a zero-order model with thermal equilibrium, however, cannot explain the color – period diagram by Eggen (1967) which leads to the establishments of two parallel first-order theories of thermal relaxation oscillation (TRO) and contact discontinuity (DSC). TRO model was advanced by Lucy (1976), Flannery (1976), and Robertson & Eggleton (1977), who suggest that contact system can not reach thermal equilibrium at dynamical equilibrium configuration and may thus undergo thermal relaxation oscillation. DSC theory was proposed by Shu and his collaborators [Shu, Lubow & Anderson 1976, 1979, Lubow & Shu 1977, also see Biermann & Thomas (1972) and Vilhu (1973) for some earlier elements of DSC model] who hypothesize contact binary can attain thermal and dynamical equilibrium but there is a temperature inversion layer in the secondary. With great attempts the so-called Kuiper's paradox and period - colour diagram may be resolved by the two different hypotheses independently. However, both theories have some difficulties to explain observations, such as the so-called W-phenomenon, i.e., the less star is hotter than the massive component (Binnendijik 1970) (see a concise summary of observations and theory by Smith 1984). Especially there are great debases between the two in nature in their simplest version (Kähler 1989). Observational studies continue and the theoretical controversy still remains (Ruciński 1997). These imply that the first-order theories of contact binary (i.e. TRO and DSC) should be improved.

The intensive disputes by the two contending schools (Lucy & Wilson 1979, Shu, Lubow & Anderson 1979) lead to an intriguing suggestion by Shu, Lubow & Anderson (1979), Shu (1980) (from theoretical viewpoint) and Wang (1995) (from the analysis of observational data) that TRO theory needs the contact discontinuity in some phases. Some observations seem to support TRO theory (Lucy & Wilson 1979, Hilditch et al 1989, Samec et al 1998). Although some criticisms about DSC model exist (Shu, Lubow & Anderson 1980), this theory is still attractive because it is successful in many aspects (Smith 1984). The contact discontinuity may be ironed out within the thermal timescale (Webbink 1977, Hazlehurst & Resfdal 1978, Papaloizou & Pringle 1979, Smith, Robertson & Smith 1980) in a steady state, however, the existence of time-dependent contact discontinuity can not be excluded because it does not violate the second law of thermodynamics (Papaloizou & Pringle 1979). However a detailed analysis is needed for this. It is highly desired to reconcile the two theories not only for removing the discrepancies but also for explaining more detailed observations (Shu 1980).

The difficulties of pure TRO and DSC models in explanation of W-phenomenon motivate us to explore the possibility to develop a second-order theory. The interaction between the secondary and the common convec-

tive envelope is thought as an important role in the W-phenomenon, Wang (1994) find that the W-phenomenon can be explained by the released gravitational energy of the secondary through its contraction corresponding to the TRO contracting phase in W-type contact systems. This is encouraging and leads to the suggestions by Wang (1995) from a sample with 32 contact systems that the A-type systems may undergo thermal relaxation oscillation with contact discontinuity whereas the contraction of secondary in W-type systems irons out this contact discontinuity.

The over-riding virtue of a contact discontinuity is that it gives a clear mechanism for making the secondary physically larger, and the primary physically smaller, than their main-sequence single-star counterparts, as is needed to satisfy the Roche-lobe filling requirements of the Kuiper paradox (Lubow & Shu 1977). On the other hand, if the system cannot be maintained in steady state by heat-carrying flows, then the capping of the radiative heat flow from the secondary by the hotter overlying common envelope should lead to an expansion of the secondary, with a resulting transfer of mass from the secondary to the primary. Such Roche-lobe overflow, from a less massive star to a more massive star to a more massive one, is known to occur slowly, so the ultimate breaking of contact caused by the expansion of the binary orbit takes a relatively long time. Once contact has been broken, however, the common envelope disappears; the secondary is no longer capped; and it will begin to shrink toward its normal single-star size. Conversely, because the larger area of the common envelope is no longer available to carry away much of the primary's interior luminosity, the primary can no longer be maintained at its suppressed contact size, and it will begin to overflow its Roche lobe. The transfer of mass from a more massive star to a less massive one is known to be unstable (e.g. Paczyński 1971), and the rapid shrinkage of the binary orbit causes the system to come into contact again. The re-establishment of the common envelope and the capping of the secondary results in its refilling its Roche lobe. Thus, would the DSC hypothesis provide the physical mechanism for the TRO hypothesis, together with a justification why the duty cycle is long for the contact phase and short for the semidetached phase, as is required by the observational statistics. The rest of this paper attempts to establish the above ideas on a more quantitative basis.

This paper is organized as following: the instability of Roche lobe and its operation in contact system are found in Sec.2; the surviving condition of DSC layer is derived from the thermodynamics in Sec. 3; and the conclusions are remarked in last sectionr.

2. THE ROCHE LOBE INSTABILITY AND DSC MODEL

It is generally believed that the two components of W UMa stars share an optically thick common convective envelope due to the dynamics equilibrium (Osaki 1965, Mochnacki 1981). The redistribution of the total luminosities (the plus of luminosities of each star), which takes place in CCE, deals with comprehensive fluid processes (Lucy 1968). Why and how to redistribute the luminosties is the main task to theoreticians. The debate of the exsiting theories of contact binaries had been attracted much consideration between 1970s and 1980s (Lucy & Wilson 1979, Shu 1980, Shu, Lubow & Anderson 1981). Shu (1980) clearly stated that the two superficially distinct theories are complementary with the crucial theoretical issue to be resolved being the secular stability of temperature inversion layer from his thought-provoking analysis. Here we argue that DSC layer is a natural results of TRO theory via the mechanism of Roche lobe instability, showing the presence of DSC layer during the expansion TRO phase.

In the following discussions we assume that the total mass and angular momentum are conserved, neglecting the spin angular momentum of two components. These assumptions are basic and the same in TRO theory, but they are unnecessary in the DSC theory. In principle, the two assumptions put more strong constraints on the theoretical model. In the conserved systems there are mainly two other parameters: mass ratio q, and mass ratio changing rate due to mass exchange $\dot{q}$, to determine the structure of the contact binaries in TRO theory. The most serious shortcoming of TRO (mentioned in the previous section) is a strong indicator that we should relax some of assumptions in TRO model. One possible way to remove this shortcoming is to supplement the interaction between CCE and the component. This inclusion may reconcile the two contending schools each other (Wang 1995).

We first show that the instability of mass exchange may prevent from the mass in the secondary being pushed into the primary through the inner Lagrangian point L_1 due to the lid effects of CCE placed on the secondary (Shu, Lubow & Anderson 1976). For a contact system with total angular momentum $\mathcal{J}$ in a circular orbit and total mass $\mathcal{M} = M_1 + M_2$, the separation between components reads

$$\mathcal{D} = \left(\frac{\mathcal{J}^2}{G\mathcal{M}^3}\right)\frac{(1+q)^4}{q^2}, \tag{1}$$

where $q = M_2/M_1$ (for the convenience we take $q \leq 1$), and G is the gravitation constant. The Roche lobe radius $R_{\rm L}$ of the secondary approximates for all mass ratio (Eggleton 1983)

$$r_{\rm L} = \frac{R_{\rm L}}{\mathcal{D}} = \frac{0.49}{0.6 + q^{\frac{2}{3}}\ln(1+q^{-\frac{1}{3}})}. \tag{2}$$

The Roche lobe of the primary will be obtained when we replace q by $1/q$. It is important to note that the Roche lobe is changing due to the mass transfer between the two components. The variation rate of the Roche lobe due to mass transfer between the two components reads

$$\frac{d\ln R_{\rm L}}{dq} = \frac{2r_{\rm L}}{3q^{\frac{1}{3}}}\left[\frac{1}{1+q^{\frac{1}{3}}} - 2\ln(1+q^{-\frac{1}{3}})\right] + \frac{2(q-1)}{q(1+q)}, \tag{3}$$

and then we have the timescale for this change with the helps of $d\ln R_{\rm L}/dt = (d\ln R_{\rm L}/dq)(dq/dt)$

$$t_{R_{\rm L}} = \left(\frac{d\ln R_{\rm L}}{dt}\right)^{-1} = f(q)t_{\rm M}, \tag{4}$$

where $t_{\rm M}$ is the timescale of mass transfer defined as

$$t_{\rm M} = \frac{M_1}{\dot{M}_1}, \tag{5}$$

here the parameter $\dot{M}_1$ is the rate of mass transfer, and the function $f(q)$ is

$$f(q) = \left\{ \frac{2(1-q)}{q} - \frac{2r_{\rm L}}{3q^{\frac{1}{3}}} \left[\frac{1}{1+q^{\frac{1}{3}}} - 2\ln(1+q^{-\frac{1}{3}}) \right] (1+q) \right\}^{-1}. \tag{6}$$

The function $f(q)$ represents the ratio of the two timescales. We have calculated the function $f(q)$ in Figure 1, showing its value for the range of q from 0.0 to 1.0. If $f(q) > 0$ then Roche lobe will expand with increases of q or shrink with decreases of q. If $f(q) < 0$ then Roche lobe will shrink with the increases of q or expand with decreases of q. It is very important to address that if $|f(q)| < 1$ then the expansion or shrinkage will be rapid than the process of mass transfer in the contact system known from equation (4). From Figure 1 the Roche lobe of the secondary expands with the increases of mass ratio. The expansion timescale is shorter than that of gaining mass from the primary until the mass ratio $q > 0.8$, indicating that the secondary is capable of swallowing more mass when $q < 0.8$. In this range of mass ratio the mass gaining is unstable. The Roche lobe of the primary will shrink due to the mass exchange. There also is an instability of mass exchange in the range of $q < 0.35$. This implies that the timescale of Roche lobe shrinkage due to mass transfer is shorter than that of mass transfer. This means that Roche lobe shrinks more rapid than the mass loss. This mass exchange instability plays important role in the structure of contact binaries. The maximum of mass ratio for the instability of the primary is 0.35. It is believed that the mass transfer will be more efficient when $q < 0.35$ with the presence of the instability of mass exchange. Thus it is expected that the mean mass ratio of A-type systems will be less than that of W-types. This is consistent with the observations. It should be noted that here we do not specify the mechanism for the energy and mass transfer. Of course the direction of mass exchange between the two components is determined by the relative potential of the star surface. Here we are trying to show the instability of mass exchange, namely, represented by $|f(q)| < 1$.

In the original DSC version the rising convective elements interior to the Roche lobe of the secondary cannot penetrate into the common convective envelope because the resulting buoyancy deficit opposes such penetration. SLA76 argued that there is a mass flow pushed by the slight excess of pressure due to a slight heating of the interior of the secondary under *constant* volume. It is very important to note that this mass flow process is based on the assumption with *constant* volume of the secondary. The mass flow was estimated by SLA79, however, their estimation follows up another assumption that *all* the energy transferred from the primary to the secondary radiates again from the secondary, neglecting the interaction between CCE and the secondary. Now we can work out a condition inhibits the returning of mass flow to the primary through the inner Lagrangian point L_1. If there is no excess pressure, then the mass flow stops. This is equivalent to $d\bar{\rho}/dt \leq 0$ if the *contact discontinuity* being lower than the temperature of CCE survives, namely, T =constant (we assume the gas is ideal), where $\bar{\rho}$ is the mean mass density within the Roche lobe of the secondary defined as $\bar{\rho} \propto M_2/R_{\rm L}^3$. We thus have

$$\frac{d\ln\bar{\rho}}{dt} \approx \frac{d}{dt}\left[\ln\left(\frac{q}{1+q}\right)\right] - \frac{3}{t_{R_{\rm L}}}, \tag{7}$$

then the condition no returning of mass flow reads

$$t_{\rm R_L} \leq 3q t_{\rm M}. \tag{8}$$

We draw the line $t_{\rm R_L} = 3qt_{\rm M}$ in Figure 1. It is obvious that all the value of $f(q)$ is always less than $3q$. This means that all the cases satisfy the condition that no mass flow returns to the primary even beyond the mass exchange instability. Therefore the assumption of *constant* volume of the secondary should be relaxed in the advanced DSC model. The contact discontinuity is time-dependent from this viewpoint at least, coinciding with that DSC layer could be maintained in a time-dependent model (Papaloizou & Pringle 1979).

3. THERMODYNAMICS OF DSC LAYER

By defining the thermal timescale as $t_{\rm Th} = \int_{M-\delta m}^{M} (4\pi r^2 \rho v_c)^{-1} dm$, Webbink(1977) first showed that the thermal diffusion time scale in the common envelope is typically of the same order as the dynamical timescale (is roughly of one orbital period). This makes the contact discontinuity disappear within one orbital period. We call the thermal diffusive process as interaction ϵ. Papaloizou & Pringle (1979) show the steady contact discontinuity violates the second law of thermodynamics, but the time-dependent contact discontinuity may exist. However in the time-dependent model it is the interaction ϵ that keeps the contact discontinuity in contact system undergoing thermal relaxation oscillation. The controversy of inner structure may be removed by this kind of interaction (Wang 1995). With the helps of the conservation of mass and momentum we can rewrite the energy equation beyond the energy generation region as

$$\rho\frac{\partial}{\partial t}(\Psi + Ts) = -\rho T\vec{v}\cdot\nabla s - \nabla\cdot\vec{F} + \epsilon, \tag{9}$$

for the inviscid fluid (e.g. Webbink 1977, and 1992 ApJ, 396, p378 for the erratum), where t is time; ρ, the density; T, the temperature; $\vec{v}$, the velocity; s, the specific entropy; $\vec{F}$, the energy flux radiated from the star; ϵ, the energy density absorbed by the secondary in the unit time due to the interaction with CCE; and Ψ, the gravitational energy per unit mass. Following the assumption by Shu, Lubow & Anderson (1980) that the specific entropy s can be decomposed in terms of a barotropic and a baroclinic one as $s = s_0(\Psi_{\rm D}) + s_1(\vec{x}, t)$, we integrate the above equation over the volume enclosed by the equipotential surfaces C and D, and obtain

$$\frac{dS}{dt} = s_0(\Psi_D)\frac{d(\Delta M)}{dt} - \iiint_{\rm CD}\frac{\rho}{T}\frac{\partial\Psi}{\partial t}dV + \frac{\epsilon}{T}\Delta V - \oint_{\rm CD}\rho s_1(\vec{x},t)\vec{v}\cdot\vec{n}dA - \iiint_{\rm CD}\frac{1}{T}\nabla\cdot\vec{F}dV, \tag{10}$$

where $S = \iiint \rho s dV$, $\Delta M = \iiint \rho dV$, $\Delta V = \iiint dV$ is the volume enclosed by the two surfaces C and D, dA is the area of the surface of contact discontinuity, and $\vec{n}$ is its normal vector. For the time-dependent case we assume

that the last two terms offset approximately as in steady case (Shu, Lubow & Anderson 1980), thus we have more physically concise form of equation (10)

$$\frac{dS}{dt} = s_0(\Psi_D)\frac{d(\Delta M)}{dt} - \iiint_{\rm CD} \frac{\rho}{T}\frac{\partial \Psi}{\partial t} dV + \frac{\epsilon}{T}\Delta V, \quad (11)$$

The first term of right hand in equation (11) represents the entropy increases due to mass exchange between CCE and the secondary, the last term does the same meanings but due to energy exchange, the second term does the entropy decreases of entropy due to the Roche lobe expansion. The enclosed volume is an open system undergoing mass and energy exchanges with its surroundings rather than an isolated volume. This equation also tells us the resulting expansion due to the interaction ϵ if the contact discontinuity layer survives: 1) if there is only exchanges of energy by thermal diffusion, namely, ΔM =const, we have

$$\iiint_{\rm CD} \rho \frac{\partial \Psi}{\partial t} dV \geq \epsilon \Delta V. \quad (12)$$

This clearly states that the surviving of contact discontinuity must be provided by the expansion. Detailed calculation should be done in the future. 2) There is a mass exchange between CCE and the secondary accompanying the energy interaction, i.e., $d(\Delta M)/dt > 0$, the expansion is at least

$$\iiint_{\rm CD} \rho \frac{\partial \Psi}{\partial t} dV \geq \epsilon \Delta V + s_0(\Psi_D) T \frac{d(\Delta M)}{dt}. \quad (13)$$

This equation predicts the secular change of orbital period due to the shift of mass ratio. 3) If the secondary keeps *constant* volume as originally suggested by Shu, Lubow & Anderson (1976), the term $d\Psi/dt = 0$, we always have $dS/dt \geq 0$ which means the discontinuity will be ironed out with the thermal diffusive timescale. The only possible way to relax this condition is the inclusion of the changes of Roche lobe. This way will permits us unifying the two contending hypotheses. According the simplest version of star structure, equations (12) and (13) will provide the expansion velocity

$$v_{\rm int} \geq R_2 \left(\frac{\epsilon}{\bar{\epsilon}}\right)\left(\frac{\Delta V}{V_2}\right) + T s_0(\Psi_{\rm D}) g_2^{-1} \frac{d(\ln \Delta M)}{dt}, \quad (14)$$

where $\bar{\epsilon} = (GM_2 \Delta M/R_2)/V_2$ is the mean density of the gravitational energy between the secondary and CCE, $g_2 = GM_2/R_2^2$ the gravitational acceleration, V_2 is the volume of the secondary. One should note that in the above estimation we neglect the down directed propagation of energy to the interior with dynamical timescale. Therefore the present estimation is somewhat higher than that of the actual. Both of the mass exchange and energy interaction result in the expansion of the secondary, we thus have the minimum velocity

$$V = \max(v_{\rm int}, v_{\rm R_L}), \quad (15)$$

where $v_{\rm R_L} = dR_{\rm L}/dt$ is the expanding velocity of the Roche lobe. This is the condition maintaining contact discontinuity. From the viewpoint of total energy (by the nuclear) conservation, the exhausted energy (i.e. ϵ) to expand the secondary lowers the re-radiating efficiency of the transferred energy from the primary. The lower temperature of the secondary than the primary may be an indicator of the presence of contact discontinuity.

4. CONCLUSIONS AND DISCUSSIONS

Introducing a discontinuity of temperature by Shu and his collaborators (1976, 1979) the thermal instability of binary (Lucy 1976, Flannery 1976) can be suppressed, but its maintenance of DSC layer opens. In this paper we try to construct the physical scenario of time-dependent model of contact binary. Only two assumptions that total mass and angular momentum of the contact system are conserved are employed in this paper. It is found that the mass exchange results in the instability of Roche lobe in some ranges of mass ratio. We show that this instability always satisfies the condition that keeps the mean density of the secondary $d\ln\bar{\rho}/dt \leq 0$. Therefore it ensures that no mass flow returns to the primary through the inner Lagrangain point L_1. The second-order theory predicts that the contact binary may be in oscillations take place about a state with a contact discontinuity. The temperature differences of DSC layer across the interface is determined by the expansion velocity.

The existing TRO and DSC theories (Lucy 1976, Flannery 1976, Robertson & Eggleton 1977; Shu, Lubow & Anderson 1976, 1979) neglect the effects of interaction ϵ between CCE and the secondary. Here we argue that it is the contact discontinuity layer that results in the interaction between CCE and the secondary and in the meanwhile it is the interaction that maintains the contact discontinuity. We find this interaction ϵ can result in some interesting issues. First it is the reason why the temperature of the secondary in A-types is lower than that of the primary. Second the maintenance of contact discontinuity needs faster mass transfer which breaks down the deep contact. Thus the shortcoming of TRO theory will be removed. It is highly desired that the unification of TRO theory and DSC model should be calculated in order to discover the nature of the contact binaries.

In the present work we do not specify the mechanism of thermal diffusion process. Although we have not performed the time-dependent model unifying TRO and DSC hypotheses, this time-dependent unified theory might give some predictions. First the secular behavior of period change of A-type systems is violent than that in W-type system in order to survive the existence of contact discontinuity. Second, the maintenance of contact discontinuity may lead to the radial oscillation of the secondary with period from a few to several ten minus. The interaction between CCE and the secondary drives such a oscillation similar to the κ-mechanism working in other types of stars. It is thus expected to find the light variation during the primary eclipse as another probe of contact discontinuity in A-type systems.

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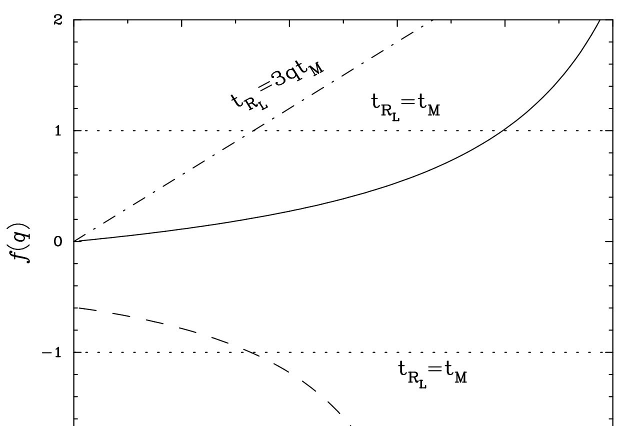

FIG. 1.— The function $f(q)$ shows the $t_{\rm R_L}/t_{\rm M}$ for the two components. The solid line represents that of the secondary, and the dashed line does the primary. See detail in the text.